

Queen Mary, University of London
MAS317 Linear Algebra II, 2007
Coursework 7

This coursework should be handed in by 4.00 pm on MONDAY November 12th in the orange box on the second floor in the mathematics building. Late coursework will not be accepted. Write your name and student number at the top and clip all the sheets together.

There are no lectures or exercise class for Linear Algebra II in Reading Week. The following course work should help you to revise the material in Ch. 3,4 and 5 on bilinear forms, congruence and symmetric matrices OVER $\mathbb{R}$, and to learn about the very similar topics of sesquilinear forms, Hermitian congruence and Hermitian matrices OVER $\mathbb{C}$ in Ch. 6.

The notes for Chapter 6 are attached. Read through them comparing the first section with (3.8)-(3.14) and (4.8), the second section with (5.1)-(5.6) and the third section with (5.8)-(5.10) and (5.13)-(5.14). Note very carefully where the differences are - complex conjugate signs are easily missed. The proofs of these results are the same as for the earlier results except for the insertion of some complex conjugate signs, and you are not asked to reproduce them (unless the questions ask you to). As you do this also work through this question sheet.

1. f is a sesquilinear form on $\mathbb{C}^2$ and the matrix of f with respect to the standard basis is $\begin{pmatrix} 2 & 1+i \\ 3 & 0 \end{pmatrix}$.
 - (i) Calculate $f((3, 1+i), (4i, 2-i))$.
 - (ii) Find the matrix of f with respect to the basis $\mathcal{B} = (1, i), (i, 1)$.
2. Write out a proof of Theorem (6.7) by copying and adapting the proof of (4.8) given in the lectures.
- 3.(a). What is the definition of *Hermitian congruence*?

Straight from the definition prove that if $\begin{pmatrix} -1 & 0 \\ 0 & 0 \end{pmatrix}$ is Hermitian congruent to A then the diagonal entries in A are real numbers and ≤ 0 .

- (b). A matrix *congruent* to a given matrix results when an elementary row operation e and the corresponding column operation e^T are applied. What is the similar result for *Hermitian congruence*?

In each of the following cases a matrix B is obtained by applying the given row and column operation to a matrix A . From the statements just given state whether it is clear that A and B are congruent, or Hermitian congruent (or both or neither).

- (i) $R_1 \rightarrow iR_1, C_1 \rightarrow iC_1,$
 - (ii) $R_1 \rightarrow R_1 + (2 + i)R_2, C_1 \rightarrow C_1 + (2 - i)C_2,$
 - (iii) $R_1 \rightarrow R_1 + 2R_2, C_1 \rightarrow C_1 - 2C_2,$
 - (iv) $R_1 \rightarrow R_1 + 2R_2, C_1 \rightarrow C_1 + 2C_2.$
4. Give the definition of an *Hermitian* matrix over $\mathbb{C}$.
Prove the following results.
- (i) In an Hermitian matrix the diagonal entries are real, and
 - (ii) If A is Hermitian and B is Hermitian congruent to A , then B is Hermitian.
5. Find the rank and signature for the following Hermitian matrices over $\mathbb{C}$ (under Hermitian congruence).

$$A = \begin{pmatrix} 1 & 2i \\ -2i & 4 \end{pmatrix}, B = \begin{pmatrix} 0 & -3i \\ 3i & 0 \end{pmatrix},$$

$$C = \begin{pmatrix} -1 & 1+i \\ 1-i & -2 \end{pmatrix}, D = \begin{pmatrix} 0 & 2+i \\ 2-i & 0 \end{pmatrix}.$$

Dr Sue McKay
2nd November 2007